OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

Mechanics 1

4728

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

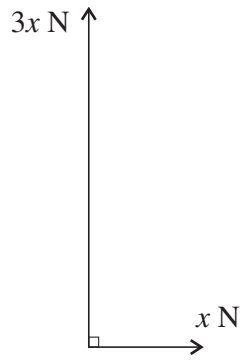
INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



Two perpendicular forces have magnitudes x N and $3x$ N (see diagram). Their resultant has magnitude 6 N.

(i) Calculate x. **[3]**

(ii) Find the angle the resultant makes with the smaller force. **[3]**

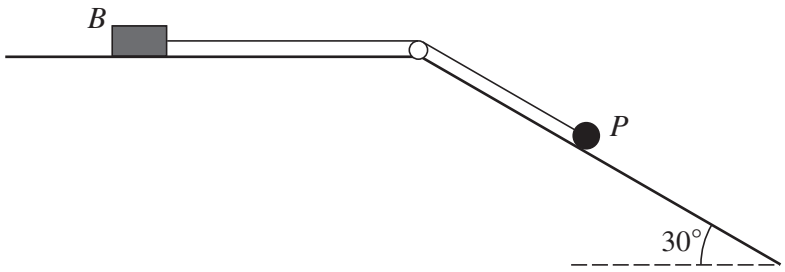
2 The driver of a car accelerating uniformly from rest sees an obstruction. She brakes immediately bringing the car to rest with constant deceleration at a distance of 6 m from its starting point. The car travels in a straight line and is in motion for 3 seconds.

(i) Sketch the (t, v) graph for the car's motion. **[2]**

(ii) Calculate the maximum speed of the car during its motion. **[3]**

(iii) Hence, given that the acceleration of the car is $2.4\,\text{m s}^{-2}$, calculate its deceleration. **[4]**

3



The diagram shows a small block B, of mass 3 kg, and a particle P, of mass 0.8 kg, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley. B is held at rest on a horizontal surface, and P lies on a smooth plane inclined at 30° to the horizontal. When B is released from rest it accelerates at $0.2\,\text{m s}^{-2}$ towards the pulley.

(i) By considering the motion of P, show that the tension in the string is 3.76 N. **[4]**

(ii) Calculate the coefficient of friction between B and the horizontal surface. **[5]**

4 An object is projected vertically upwards with speed $7\,\text{m s}^{-1}$. Calculate

(i) the speed of the object when it is 2.1 m above the point of projection, **[3]**

(ii) the greatest height above the point of projection reached by the object, **[3]**

(iii) the time after projection when the object is travelling downwards with speed $5.7\,\text{m s}^{-1}$. **[3]**

5 **(i)**

Fig. 1

A particle P of mass 0.5 kg is projected with speed $6\,\text{m s}^{-1}$ on a smooth horizontal surface towards a stationary particle Q of mass m kg (see Fig. 1). After the particles collide, P has speed $v\,\text{m s}^{-1}$ in its original direction of motion, and Q has speed $1\,\text{m s}^{-1}$ more than P. Show that $v(m + 0.5) = -m + 3$. **[3]**

(ii)

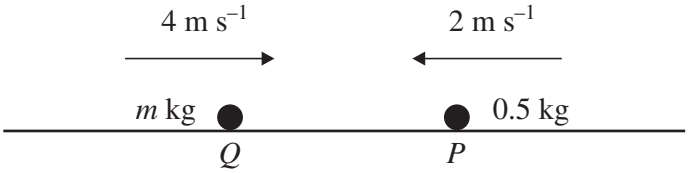


Fig. 2

Q and P are now projected towards each other with speeds $4\,\text{m s}^{-1}$ and $2\,\text{m s}^{-1}$ respectively (see Fig. 2). Immediately after the collision the speed of Q is $v\,\text{m s}^{-1}$ with its direction of motion unchanged and P has speed $1\,\text{m s}^{-1}$ more than Q. Find another relationship between m and v in the form $v(m + 0.5) = am + b$, where a and b are constants. **[4]**

(iii) By solving these two simultaneous equations show that $m = 0.9$, and hence find v. **[4]**

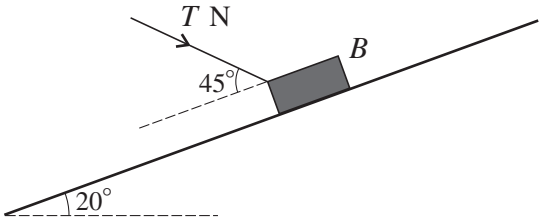
[Questions 6 and 7 are printed overleaf.]

6 A block B of weight 10 N is projected down a line of greatest slope of a plane inclined at an angle of 20° to the horizontal. B travels down the plane at constant speed.

(i) (a) Find the components perpendicular and parallel to the plane of the contact force between B and the plane. **[2]**

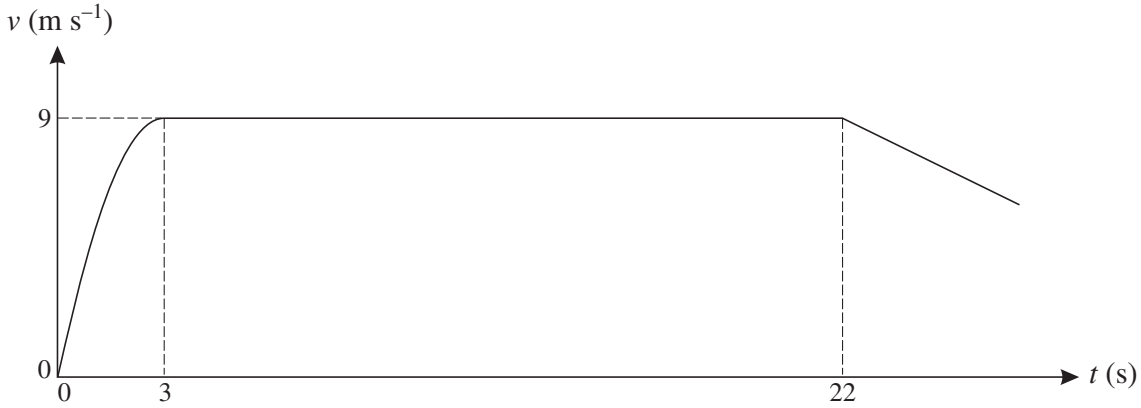
(b) Hence show that the coefficient of friction is 0.364, correct to 3 significant figures. **[2]**

(ii)



B is in limiting equilibrium when acted on by a force of T N directed towards the plane at an angle of 45° to a line of greatest slope (see diagram). Given that the frictional force on B acts down the plane, find T. **[7]**

7



A sprinter S starts from rest at time $t = 0$, where t is in seconds, and runs in a straight line. For $0 \leqslant t \leqslant 3$, S has velocity $(6t - t^2)\,\mathrm{m\,s^{-1}}$. For $3 < t \leqslant 22$, S runs at a constant speed of $9\,\mathrm{m\,s^{-1}}$. For $t > 22$, S decelerates at $0.6\,\mathrm{m\,s^{-2}}$ (see diagram).

(i) Express the acceleration of S during the first 3 seconds in terms of t. **[2]**

(ii) Show that S runs 18 m in the first 3 seconds of motion. **[5]**

(iii) Calculate the time S takes to run 100 m. **[3]**

(iv) Calculate the time S takes to run 200 m. **[7]**

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1 i	$x^2 + (3x)^2 = 6^2$ $10x^2 = 36$ $x = 1.9(0)$ (1.8973..)	M1 A1 A1 [3]	Using Pythagoras, 2 squared terms May be implied Not surd form unless rationalised $(3\sqrt{10})/5$, $(6\sqrt{10})/10$
ii	$\tan\theta = 3x/x$ (= 3×1.9/1.9) = 3 $\theta = 71.6^o$ (71.565..)	M1 A2 [3]	Must target correct angle. Accept $\sin\theta = 3\times1.9/6$ or $\cos\theta = 1.9/6$ which give θ=71.8°, θ=71.5° respectively, A1. **SR** $\theta = 71.6^o$ from $\tan\theta = 3x/x$ if x is incorrect; x used A1, no evidence of x used A2
2 i		B1 B1 [2]	Inverted V shape with straight lines. Starts at origin, ends on t-axis, or horizontal axis if no labelling evident
ii	$6 = 3v/2$ $v = 4$ ms^{-1}	M1 A1 A1 [3]	Not awarded if special (right angled, isosceles) triangle assumed, or $s = (u+v)t/2$, or max v at specific t.
iii	T accn = 4/2.4 or s accn = 16/(2x2.4) T accn = 1 2/3 s or s accn = 10/3 Deceleration = 4/(3 - 1 2/3) or 16/2(6-10/3) Deceleration = 3 ms^{-2}	M1* A1 D*M1 A1 [4]	Uses $t = v/a$ or $s = v^2/2a$. May be implied Accept 4/(3 - 1.67) or 16/2(6-3.33) Accept 3.01; award however $v = 4$ obtained in (ii). $a = -3$ gets A0.
3 i	0.8gsin30 0.8x0.2 $0.8\times9.8\sin30 - T = 0.8\text{x}0.2$ $T = 3.76$ N AG	B1 B1 M1 A1 [4]	Not for 3.92 stated without justification Or 0.16 Uses N2L // to slope, 3 non-zero terms, inc ma Not awarded if initial B1 withheld.
ii	$3.76 - F = 3\times0.2$ $F = 3.16$ $3.16 = \mu\text{x}3\times9.8$ $\mu = 0.107$ (0.10748)	M1 A1 A1 M1 A1 [5]	Uses N2L, B alone, 3 non-zero terms Needs *correct value* of T. May be implied. Uses $F=\mu R$ (Accept with $R = 3$, but not with R=0.8g(cos30), F=0.6, F=3.76, $F = f$(mass P)) Not 0.11, 0.108 (unless it comes from using g=9.81 consistently through question.

4 i	$v^2 = 7^2 - 2\times9.8\times2.1$ $v = 2.8$ ms^{-1}	M1 A1 A1 [3]	Uses $v^2 = u^2 - 2gs$. Accept $7^2 = u^2 + 2gs$
ii	$v = 0$ $0^2 = 7^2 - 2\times9.8s$ $s = 2.5$ m	B1 M1 A1 [3]	Velocity = 0 at greatest height Uses $0 = u^2 - 2gs$. Accept $7^2 = 2\times9.8s$.
iii	$v = -5.7$ (or $t = 0.71$ oef to reach greatest height) $-5.7 = 7 - 9.8t$ or $5.7 = (0+) 9.8T$ $t = 1.3(0)$ s (1.2959..)	B1 M1 A1 [3]	Allows for change of direction Uses $v = u +$ or $- gt$. Not 1.29 unless obtained from g=9.81 consistently
5 i	$0.5\times6 = 0.5v + m(v+1)$ $3 = 0.5v + mv + m$ $v(m + 0.5) = -m + 3$ AG	M1 A1 A1 [3]	Uses CoLM. Includes g throughout MR-1
ii	Momentum before = +/- $(4m - 0.5\times2)$ +/- $(4m - 0.5\times2) = mv + 0.5(v+1)$ $4m - 0.5\times2 = mv + 0.5(v+1)$ $v(m+0.5) = 4m - 1.5$	B1 M1 A1 A1 [4]	Includes g throughout MR-1 Needs opposite directions in CoLM on "before" side only. RHS in format am + b or b + am. Ignore values for a and b if quoted.
iii	$4m - 1.5 = - m + 3$ $5m = 4.5$ $m = 0.9$ kg AG $0.9 + v(0.9+0.5) = 3$ or $4\times0.9 - 1.5 = v(0.9+0.5)$ $v = (3-0.9)/(0.9+0.5) = 2.1/1.4$ $v = 1.5$ ms^{-1}	M1 A1 M1 A1 [4]	Attempts to obtain eqn in 1 variable from answers in (i) and (ii) Ignore $m = -0.5$ if seen Substitutes for m=0.9 in any m, v equation obtained earlier.
6 ia	Perp = 10cos20 (= 9.3967 or 9.4) // = 10sin20 (= 3.4202)	B1 B1 [2]	Includes g, MR -1 in part (i). Accept –ve values.
b	$\mu = 10\sin20/10\cos20 = \tan20$ (= 3.42/9.4) $\mu = 0.364$ (0.36397..) AG	M1 A1 [2]	Must use $\lvert F\rvert = \mu\lvert R\rvert$ Accept after inclusion of g twice
ii	*No misread, and resolving of 10 and T required* $R = 10\cos20 + T\cos45$ $F = T\cos45 - 10\sin 20$ or $T\cos45 = \mu R + 10\sin20$ $T\cos45 - 3.42 = 0.364(9.4 + T\cos45)$ $0.707T - 3.42 = 3.42 + 0.257T$ $0.45T = 6.84$ $T = 15.2$ N (15.209..)	M1* A1 M1* A1 D*M1 A1 A1 [7]	3 term equation perp plane, 2 unknowns $9.4 + 0.707T$ (accept $9.4+.71T$) 3 term equation // plane, 2 unknowns $0.707T - 3.42$ (accept $0.71T - 3.4$) Substitutes for F and R in F=0.364R *Award final A1 only for T = 149 N after using 10g for weight*

7 i	$a = \mathrm{d}v/\mathrm{d}t$ $a = 6 - 2t$ ms^{-2}	M1 A1 [2]	Differentiation attempt. Answer 6-t implies division by t
ii	$s = \int v\mathrm{d}t$ $s = \int 6t - t^2 \ \mathrm{d}t$ $s = 3t^2 - t^3/3$ (+c) $t = 0$, $v = 0$, $c = 0$ $t = 3$, $s = 3\text{x}3^2 - 3^3/3$ $s =$ 18 m AG	M1* A1 B1 D*M1 A1 [5]	Integration attempt on v Award if limits 0,3 used Requires earlier integration Does not require B1 to be earned.
iii	Distance remaining (= 100 -18) = 82 Total time = 3 + 82/9 $T = 12.1$ s (12 1/9)	B1 M1 A1 [3]	 Numerator not 100 Not 109/9
iv	Distance before slows = 18 + (22 - 3)x9 Distance while decelerating = 200 - 189 = 11 $11 = 9t - 0.3t^2$ or $11 = (9+8.23)t/2$ or $8.23 = 9 - 0.6t$ $t = 1.28$ (1.2765.., accept 1.3) $T = 23.3$ s (23.276..)	M1* A1 D*M1 A1 D*M1 A1 A1 [7]	(=189 m) Two sub-regions considered Accept 10.99. 10.9 penalise -1PA. Uses $s = ut - 0.5\text{x}0.6t^2$, or $v^2 = u^2 - 2\text{x}0.6s$ with $s=(u+v)t/2$ or $v=u+at$ Finds t. (If QE, it must have 3 terms and smaller positive root chosen.)